INEQUALITY MEASURES

MIERNIKI NIERÓWNOŚCI

Łukasz Jabłoński[1]

UNIWERSYTET EKONOMICZNY W KRAKOWIE,
UNIVERSITY OF ECONOMICS FACULTY OF COMMODITY SCIENCE AND PRODUCT MANAGEMENT DEPARTMENT OF APPLIED ECONOMICS

Abstract: The purpose of the article is to present ways to measure inequality. The article describes the last three ways to measure inequality, i.e. based on the Lorenz curve, generalized entropy and the social welfare function. It omits, however, the share and division measures, i.e. the absolute and relative differences, variance, quartile and standard deviations, or positional inequality measures based on percentiles, deciles, quartiles, quantiles, the coefficient of variation and the McLoone index, which results from the fact that although they are often used in empirical research, they illustrate division rather than inequality sensu stricto (i.a. Park 1984, pp. 42-44; Heshmati 2004; Jóźwiak, Podgórski 2012; Jabkowski 2009, pp. 32-35). Thus, the article does not include a wider presentation of these measures.

Streszczenie: Celem artykułu jest przedstawienie sposobów pomiaru nierówności. W artykule opisano trzy ostatnie sposoby pomiaru nierówności, tj. w oparciu o krzywą Lorenza, uogólnioną entropię i funkcję dobrobytu. Pomija jednak miary akcji i podziału, tj. różnice bezwzględne i względne, wariancje, odchylenia kwartylowe i standardowe czy też mierniki nierówności pozycji oparte na percentylach, decylach, kwartylach, kwartylach, kwantylach, współczynniku zmienności i indeksie McLoone, co wynika z faktu, że choć są one często wykorzystywane w badaniach empirycznych, to jednak ilustrują raczej podział, a nie zmysłową nierówność sensu stricto (i.a. Park 1984, s. 42-44; Heshmati 2004; Jóźwiak, Podgórski 2012; Jabkowski 2009, s. 32-35). W związku z tym artykuł nie zawiera szerszej prezentacji tych środków.

Keywords: inequality measures, Lorenz curve, generalized entropy measures, welfare function.

Słowa kluczowe: mierniki nierówności, krzywa Lorenza, ogólne mierniki entropii, funkcja dobrobytu.

[1] Łukasz Jabłoński, dr, Uniwersytet Ekonomiczny w Krakowie, Katedra Zarządzania Jakością, e-mail: lukaszj@uek.krakow.pl.

Introduction

The literature provides a number of measures which *"make it possible to present in quantitative terms such a complex phenomenon as inequality"* (Ulman, Wałęga, 2006, p. 79). The inequality category is positive and normative by nature. The positive or rather instrumental character of inequality results from the fact that it always describes a scale of unequal access to 'goods and values' between the members of a given human community. On the other hand, the normative nature of inequality relates to the fact that it is often accompanied by an ethical norm, i.e. the principle of justice, freedom (Kot, 2004, pp. 45-46). For this reason, the measures of inequality should not only be of instrumental character, which seems obvious for any measure, but they should also reflect value judgments.

U. Ebert (2009, p. 555) explains that the tradition of reflecting the normative dimension in inequality measures was started by A.C. Pigou (1912) and Dalton (1920). Half a century later, A.B. Atkinson (1970), S.C. Kolm (1976a, b) and A.K. Sen (1973) formulated the foundations of (general) normative theory in this research area, and thus also the axioms (conditions) of measures of this economic category. They result from theoretical foundations of inequality studies and empirical requirements, e.g. the need to capture by an inequality measure any change in the distribution of income (or other goods) in the entire population (Jabkowski, 2009, p. 26).

The literature highlights a few basic criteria (conditions) for assessing inequality measures, i.e.[2]:

- Condition 1: the Pigou-Dalton transfer principle, which means that the transfer of income between persons with different income levels should be reflected in the level of inequality measure. Its level should, therefore, increase as a result of the transfer of income (or other goods) from a less to a more affluent person and decrease with the reduction in the disproportion between incomes in the studied population.
- Condition 2: income scale independence, which means that the value of inequality measure cannot change depending on the measurement scale used for particular goods[3].

[2] Among others (Park 1984, p. 36-38; Kot 2000, p. 118-121; Cowell 2009, p. 60-65; Jabkowski 2009, p. 27-30; Ebert 2009, p. 555-574). For example S.C. Kolm (1976a, p. 426) lists eight conditions, which detail those presented below.

[3] This condition applies to two issues. Firstly, when measurement of income (or other goods) takes place on a different scale, e.g. different currencies (USD, EUR, PLN), and secondly, when the value of income (or other goods) changes in the same proportion for each person or their groups in a given population. That is why, measures of volatility (variance), frequently used in empirical research, are not good measures of inequality. They are vulnerable to the units (scales) in which the value of income is measured. For this reason, calculation of income variance gives different inequality measures depending on the adopted currency.

- Condition 3: principle of population, which means that the value of inequality measure should not change in case of a k-fold increase of the value characterised by the same value and distribution of income (or other goods).
- Condition 4: decomposability, which means that the value of inequality measure in the entire population should be correlated with the values of inequality measures calculated for any of its subgroups.

In the literature, there are several inequality measures, which as explained by K.H. Park (1984, p. 38), can be classified as belonging to one of the four categories, i.e.[4]:

1. Share and division measures, i.e. absolute and relative differences, variance, quartile and standard deviations, and also positional inequality measures based on percentiles, deciles, quartiles, quantiles, the coefficient of variation and the McLoone index.
2. Measures based on the M.O. Lorenz (1905) curve, which include the Gini coefficient, relative mean deviation; the measure of R.R. Shultz (1951) and S. Kuznetz (1957), as well as the measure of Kakwani (1980);
3. Measures based on generalized entropy, which include, among other things, the Theil index (Theil's T and L);
4. Measures based on the function of social welfare, i.e. the Dalton, Atkinson and Ebert measures.

Therefore, the purpose of the article is to present ways to measure inequality. The article describes the last three ways to measure inequality, i.e. based on the Lorenz curve, generalized entropy and the social welfare function. It omits, however, the share and division measures, i.e. the absolute and relative differences, variance, quartile and standard deviations, or positional inequality measures based on percentiles, deciles, quartiles, quantiles, the coefficient of variation and the McLoone index, which results from the fact that although they are often used in empirical research, they illustrate division rather than inequality *sensu stricto* (i.a. Park 1984, pp. 42-44; Heshmati 2004; Jóźwiak, Podgórski 2012; Jabkowski 2009, pp. 32-35). Thus, the article does not include a wider presentation of these measures.

[4] A.K. Sen (1973) divides inequality measures into two groups, i.e. positive (instrumental) and normative ones. The positive measures quantify income inequality in an objective manner by applying measures of statistical division (dispersion). The normative measures, on the other hand, use the function of social welfare, which allows to observe a decrease in the welfare resulting from unequal distribution of goods. That is why, positive measures describe the distribution of goods (e.g. income, consumption) in the population without any reference to value judgments. However, making a restrictive distinction between the positive and normative division and inequality measures arouses much controversy.

1. Measures based on the Lorenz curve

The inequality measures based on the Lorenz curve are constructed without imposing the functional form of statistical distribution of income differences in a given population.

The most popular inequality measure based on the Lorenz curve is the Gini coefficient, which is characterised by a simple and intuitive graphic interpretation. In geometric terms, it illustrates the ratio of the area between the line representing a perfectly even distribution and the Lorenz curve to the entire area under the line representing a perfectly even distribution. Thus, the Gini coefficient (G) can be written as follows (Park 1984, p. 38; Jabkowski 2009, pp. 35, 37)[5]:

$$G = \frac{\frac{1}{n}(n-1)\sum_{i=1}^{n}\sum_{j=1}^{n}\left|y_i - y_j\right|}{2\overline{y}} \tag{1}$$

where: n – population size;
y_i, y_j – value of given goods (e.g. income) in possession of i-th and j-th individual or their groups;
$\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ – average income value for all individuals in the population.

The Gini coefficient assumes values from the range of <0, 1>, where 0 means perfect equality, while 1 – perfect inequality in income distribution between representatives of the population. It is worthwhile to emphasise that this coefficient meets the Pigou-Dalton transfer principle (condition 1), income scale independence (condition 2) and the principle of population (condition 3), as well as a weak version of condition (4) decomposability (Jabkowski 2009, p. 37)[6]. For this reason, the Gini coefficient is often used in empirical research on inequality. However, the basic weakness of the Gini coefficient results from the fact that its value is more sensitive to income transfers that occur between individuals achieving the near average income for the entire population than in the marginal,

[5] P. Jabkowski (2009, p. 36) points out that there are several ways to analytically capture the Gini coefficient, both for discrete and continuous distributions.

[6] This means that the value of the Gini coefficient for the entire population is different than the sum of the values of this coefficient calculated for particular parts of the population.

i.e. the upper and lower ranges of distribution (Park 1984, p. 37; Slottje, Raj 1998, p. 7)[7].

The Lorenz curve is also used to construct the relative mean deviation measure developed by C. Bresciani-Turroni (1910) having the following form (Park 1984, p. 39):

$$R = \frac{\frac{1}{n}\sum_{i=1}^{n}\left|y_i - \overline{y}\right|}{2\overline{y}}. \tag{2}$$

The R measure is the arithmetic mean of absolute deviations of income of an individual or a group of individuals (y_i) from the average value ($\overline{y}$), divided by twice the value of average income for the entire population (n) and assumes the value within the range of <0, 1>. It is worthwhile to emphasise that although interpretation of the value of this index differs from the Gini coefficient, it is more intuitive from the economic perspective. The value of this index informs of the percentage of the total income of the population, which should be shifted from the group of individuals with higher than average income to the groups of persons with lower than average income, so that the average income achieved by both groups is equal (Kakwani 1980).

The Lorenz curve has been also used by R.R. Schultz (1951) and S. Kuznetz (1957). The R.R. Schultz inequality index is calculated by summing up the differences that occur between the slope of equal distribution line, which equals 1, and the slope of the Lorenz curve in its various points. On the other hand, the S. Kuznets measure is calculated as a quotient of absolute deviations between percentage shares of population and percentage shares of income and the size of the population. N.C. Kakwani (1980) has proved that the Schultz and Kuznets indices are characterised by the same analytical form[8] and economic interpretation of their values as the relative mean deviation measures (Park 1984, p. 40). Thus, they can be determined on the basis of equation (2). However, the relative mean deviation measures and

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7 N.C. Kakwani (1980) proposed a more generalised form of the Gini coefficient (G_α), which is more sensitive to goods transfers in the marginal, i.e. the upper and lower ranges of distribution. It can be written as $G_\alpha = \left[n - 1\middle/ n\left(\sum_{i=1}^{n} i^\alpha - n\right)\right]\frac{1}{\overline{y}}\sum_{i=1}^{n}\left(\overline{y} - y_i\right)\left(n + 1 - i\right)^n$, where: i reflects income units, α – parameter of sensitivity of G_α coefficient to goods transfers between individuals that achieve income. When $\alpha > 1$, then the lower distribution range is more sensitive to transfers, however, when $\alpha < 1$, then the upper range of distribution becomes more sensitive to transfers. In case when $\alpha = 1$, then G_α assumes the form of a standard Gini coefficient. Modifications to the Gini coefficient so that it was more sensitive to transfers in the lower and upper ranges of distribution were carried out, among others, by: (Donaldson, Weymark 1980; Yazhaki 1983; Chotikapanich, Griffiths 2001). Other weaknesses of the Gini coefficient have been described *inter alia* by (Kołodko 2014, pp. 27-30).

8 Therefore, the analytical forms of Schultz and Kuznets inequality measures are similar to formula (2).

the Schultz and Kuznets measures do not meet the criterion of the Pigou-Dalton transfer principle (condition 1), and because of that they are not commonly used in empirical research.

On the basis of the Lorenz curve, N.C. Kakwani (1980) also constructed an inequality measure, which can be written as (Slottje, Raj 1998, p. 8):

$$K = \frac{\overline{\ell} - \sqrt{2}}{2 - \sqrt{2}}, \tag{3}$$

where: $\overline{\ell}$ is the length of the Lorenz curve, calculated as:

$\overline{\ell} = \sum_{k=1}^{n} \overline{\ell}_k$, while $\overline{\ell}_k = \sqrt{q_k^2 + \frac{1}{n^2}}$, whereby q_k reflects the share of k-th (e.g. decyl) group of individuals in the income value characteristic for the entire population of individuals (n).

This index assumes values within the range of <0, 1>, where 0 means perfect equality, while 1 – perfect inequality between individuals in a given population in relation to the value of specific goods (e.g. income)[9]. In addition, it has been proven that the index meets the income scale independence criterion (condition 2) and the principle of population criterion (condition 3), as well as the Pigou-Dalton transfer principle (condition 1). However, N.C. Kakwani (1980) also proved that the K index is more than the Gini coefficient sensitive to goods transfers in the marginal, i.e. the upper and lower income ranges (Park 1984, p. 39).

The main advantage of inequality measures based on the Lorenz curve lies in the intuitive economic interpretation of their values. They meet the criteria set to inequality measures, in particular conditions 1-3. Their limited usability for empirical research results, however, from the fact that they only to a limited extent meet the decomposability criterion (Foster, Shneyerov 1999, pp. 89-90)[10]. This criterion is particularly important for the studies of inequality in populations differentiated by specific characteristics, i.e. among others the geographical location, race, gender, etc. Thus, the key issue in such studies is to estimate what part of the overall inequality results from intra-group and inter-group variations.

9 If each representative of the population achieves the same value of specific goods (e.g. income), then the length of the Lorenz curve ($\Sigma\iota_k$) equals $2^{1/2}$, i.e. the length of the equal distribution line. If, on the other hand, a single individual has the total value of specific goods characteristic for the entire population, then the length of the Lorenz curve equals 2. Therefore, given the range for determining the length of the Lorentz curve, i.e. <$2^{1/2}$, 2>, the Kakwani index assumes the value between 0 and 1.

10 P. Ulman and A. Wałęga (2006, p. 79) state that the Gini coefficient is not "*easily decomposed into subgroups of the studied population. In other words, we are unable (...) to answer the question to what extent a given subgroup is 'responsible' for the overall level of social inequality.*

2. Generalized entropy measures

The inequality measures based on generalized entropy (hereinafter: the entropy measures), a general class of which was presented by A.F. Shorrocks (1980), include parameter α, which determines the distance between income values in various distribution ranges. Parameter α can adopt positive values and determines the sensitivity of entropy measures to changes in the distribution of income in various, particularly lower and upper, distribution ranges[11]. Therefore, at lower (higher) values of parameter α, the levels of entropy indices are more sensitive to changes in the income distribution in the lower (upper) distribution ranges. Thus, the class of inequality measures based on generalized entropy *GE*(α) can be generally written as follows (Foster, Shneyerov 1999, p. 94):

– for α = 0, the generalized entropy index takes the form of the Theil's *L* index (*L*), i.e.:

$$GE(0) = L = \frac{1}{n}\sum_{i=1}^{n} \ln\left(\frac{\bar{y}}{y_i}\right); \tag{4}$$

– for α = 1, the generalized entropy index takes the form of the Theil's *T* index (T), i.e.:

$$GE(1) = T = \frac{1}{n}\sum_{i=1}^{n} \frac{y_i}{\bar{y}} \ln\left(\frac{y_i}{\bar{y}}\right); \tag{5}$$

– for α ≠ 0 and α ≠ 1, the generalized entropy index takes the following form:

$$GE(\alpha) = \frac{1}{\alpha(\alpha-1)}\frac{1}{n}\sum_{i=1}^{n}\left[\left(\frac{y_i}{\bar{y}}\right)^{\alpha} - 1\right], \tag{6}$$

where:
α – parameter describing the weight assigned to the distance between the values of the goods (e.g. income) in various distribution ranges;
n – population size;
y_i – the value of income of i-th individual or a group of such individuals;
$\bar{y}$ – average income value for the population.

With equal distribution of goods (e.g. income) in the population, the inequality indices based on generalized entropy *GE*(α) (4)-(6) assume value 0, and in case of extreme inequality, the value of these indices depends on parameter α, i.e.:

– for α = 0, $GE(0) = L = \frac{\ln(1/n)}{n}$; (7)

[11] This property is the source of advantages of the entropy measures as compared to the measures based on the Lorenz curve, whose level is sensitive to changes in the distribution of goods between individuals near the average.

– for α = 1, $GE(1) = T = \ln(n)$; (8)

– for α ≠ 0 and α ≠ 1, $GE(\alpha) = \frac{1}{\alpha(\alpha-1)}\left(n^{\alpha-1} - 1\right)$. (9)

The wide application of inequality measures based on generalized entropy results from the possibilities of their additive decomposition (condition 4). Thus, these measures can be used to decompose general inequalities into k-th subgroups, determining the share of such subgroups in the description of overall inequality (Ulman, Wałega 2009, p. 80).

J.E. Foster and A.A. Shneyerov (1999, p. 94) point out that each entropy measure, that is from the class of so called generalised entropy *GE*(α), can be additively decomposed with the use of decomposition coefficient in the form of:

$$w_{k(\alpha)} = \left(\frac{n_k}{n}\right)\left(\frac{\overline{y}_k}{\overline{y}}\right)^{\alpha} \quad (10)$$

where: n_k – the size of the *k*-th subgroup in *n*-element population;
$\overline{y}_k$ – average income value in the *k*-th subgroup;
$\overline{y}$ – average income value for *n*-element population.

Thus, in general terms, entropy measures *GE*(*α*) in the forms (4-6) can be decomposed in the following manner, i.e.:

$$GE(\alpha) = \sum_{j=1}^{k} w_{k(\alpha)} GE_W^k(\alpha) + GE_B(\alpha), \quad (11)$$

where: $GE_W^k(\alpha)$ – generalized entropy inequality coefficient calculated for the *k*-th subgroup (intra-group) at a given α value;
$GE_B(\alpha)$ – generalized entropy inequality coefficient calculated on the basis of average values for particular groups (inter-group).

Given the different values of parameter α decomposition equations for entropy measures *GE*(α) can be written as:

– for α = 0, $GE(0) = L = \sum_{j=1}^{k} w_{k(0)} L_W^k + L_B$, where: $w_{k(0)} = \frac{n_k}{n}$; (12)

– for α = 1, $GE(1) = T = \sum_{j=1}^{k} w_{k(1)} T_W^k + T_B$, where: $w_{k(1)} = \frac{n_k}{n}\frac{\overline{y}_k}{\overline{y}}$; (13)

– for α ≠ 0 and α ≠ 1, $GE(\alpha) = \sum_{j=1}^{k} w_{k(\alpha 1)} GE_W^k(\alpha) + GE_B(\alpha)$, (14)

at $w_{k(\alpha)} = \left(\frac{n_k}{n}\right)\left(\frac{\overline{y}_k}{\overline{y}}\right)^{\alpha}$, where: L_W^k, T_W^k, $GE_W^k(\alpha)$ – intra-group entropy measures for k-th subgroup in the population at α value equal α = 0, α = 1 and α ≠ 0 and α ≠ 1, respectively; L_B, T_B, $GE_B(\alpha)$ – inter-group entropy measures calculated on the basis of average values for particular groups at α value equal α = 0, α = 1 and α ≠ 0 and α ≠ 1, respectively; $w_{k(0)}$, $w_{k(1)}$, $w_{k(\alpha)}$ – decomposition coefficients determined on the basis of equation (10).

Taking into account various variants of decomposition coefficients $w_{k(\alpha)}$ (equation 10) and entropy measures for various values of parameter α, namely L, T, $GE(\alpha)$ (equations 4-6), the extended forms of decomposition equations can be written as (Shorrocks 1980):

– for α = 0:

$$L = \underbrace{\sum_{k=1}^{K} \frac{n_k}{n} L^k}_{\text{intra-group inequality}} + \underbrace{\sum_{k=1}^{K} \frac{n_k}{n} \ln\left(\frac{\overline{y}}{\overline{y}_k}\right)}_{\text{inter-group inequality}} \tag{15}$$

– for α = 1:

$$T = \underbrace{\sum_{k=1}^{K} \frac{n_k \overline{y}_k}{n\overline{y}} T^k}_{\text{intra-group inequality}} + \underbrace{\sum_{k=1}^{K} \frac{n_k \overline{y}_k}{n\overline{y}} \ln\left(\frac{\overline{y}_k}{\overline{y}}\right)}_{\text{inter-group inequality}} \tag{16}$$

– for α ≠ 0 and α ≠ 1:

$$GE(\alpha) = \underbrace{\sum_{k=1}^{K} \left(\frac{n_k}{n}\right)\left(\frac{\overline{y}_k}{\overline{y}}\right)^{\alpha} GE_W^k(\alpha)}_{\text{intra-group inequality}} + \underbrace{\frac{1}{\alpha(\alpha-1)}\left[\sum_{k=1}^{K}\left[\left(\frac{n_k}{n}\right)\left(\frac{\overline{y}_k}{\overline{y}}\right)^{\alpha}\left(\frac{\overline{y}_k}{\overline{y}}\right)^{\alpha} - 1\right]\right]}_{\text{inter-group inequality}}. \tag{17}$$

Thus, the first expressions in equations (15)-(17) describe inequalities within k-th subgroups in the n-element population, and the other ones – inequalities between the separated subgroups in the population. In turn, if parameter α = 2, then the entropy measure assumes the form of one-half of the square of the coefficient of variation (VC), which is also a commonly used inequality measure, i.e. (Foster, Shneyerov 1999, p. 94):

$$GE(2) = \frac{1}{2}(VC)^2 = \frac{\frac{1}{n}\sum_{i=1}^{n}\left(\ln y_i - \ln \overline{y}\right)^2}{2\left(\ln \overline{y}\right)^2}, \tag{18}$$

where: $\ln \overline{y} = \frac{1}{n}\sum_{i=1}^{n} \ln y_i$ is the geometric mean of y_i variable distribution.

The measure in form (18) meets the condition of decomposability. Thus, this measure can be decomposed in a manner resulting from equation (14).

3. Measures based on the welfare function

The inequality measures based on the Lorenz curve and the entropy measures are of instrumental, i.e. positive, character. Popular measures of inequality, classified as normative measures, are the indices of H. Dalton (1920) and A.B. Atkinson (1970), which are based on the social welfare function (Park 1984, p. 41).

The Dalton index is based on utilitarian assumptions. Therefore, firstly, social welfare is a sum of $u(\overline{y})$ the individual utility levels $\sum_{i=1}^{n} u(y_i)$, resulting from the achieved income, and secondly, the n-element population encounters identical utility functions with diminishing marginal utility. Therefore, the social welfare reaches its maximum with an equal distribution of income, while the proportional loss of welfare resulting from income inequality can be written using the Dalton index (D) in the form:

$$D = 1 - \frac{\sum_{i=1}^{n} u(y_i)}{nu(\overline{y})}. \tag{19}$$

With equal distribution of income, the sum of individual utility levels equals n-fold the average utility for the population, $\sum_{i=1}^{n} u(y_i) = nu(\overline{y})$, while the Dalton index equals zero ($D = 0$). In turn, with unequal distribution of income, the n-fold of average utility for the population is greater than the sum of individual utility levels, and then $nu(\overline{y}) > \sum_{i=1}^{n} u(y_i) > 0$ the Dalton index (D) is within the range of (0, 1). What is more, the higher level of inequality is reflected with the higher value of the Dalton index.

A.F. Cowell (2009) developed the Dalton index to include the parameter reflecting inequality aversion, $\varepsilon > 0$ to the form:

$$D_\varepsilon = 1 - \frac{\frac{1}{n}\sum_{i=1}^{n}\left(y_i^{1-\varepsilon} - 1\right)}{\overline{y}^{1-\varepsilon} - 1}, \ \varepsilon > 0. \tag{20}$$

With $\varepsilon > 0$ there is a social preference for equal distribution of income. The higher the value of parameter ε, the higher weight is assigned to transfers in the lower range and the lower weight to those in the upper range of income distribution. There are two limits of parameter ε. When parameter ε tends to infinity ($\varepsilon \to \infty$), then the Dalton measure (D_ε) is of the Rawlsian character, which means that social welfare depends on the income of the poorest portion of the society, i.e. lower distribution ranges. On the other hand, when $\varepsilon \to 0$, then welfare is a linear function in relation to income and its distribution does not impact the value of the welfare index (D_ε).

Because of the weaknesses of the Dalton index (D, D_ε)[12], A.B. Atkinson (1970) developed another measure of inequality based on the welfare function and taking into account the risk aversion in the society (ε). The A.B. Atkinson measure is based on equally distributed equivalent income[13], $y_e = \sum_{i=1}^{n} u(y_i)$, and can be written as:

$$A_\varepsilon = 1 - \frac{1}{\bar{y}} \left[\frac{1}{n} \sum_{i=1}^{n} (y_i)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{21}$$

If income is equally distributed in the society, then $y_e = \bar{y}$, and the value of the Atkinson index equals zero. If, however, the total income is in possession of a single person, then the equivalent income reaches zero, and the value of the Atkinson index is 1. Therefore, with unequal distribution of income, the value of the Atkinson index is within the range of (0, 1). What is more, the difference between the value of equivalent income and average income can be interpreted as loss of welfare resulting from inequality. Thus, the Atkinson index illustrates the loss of welfare as a result of inequality as percentage of average income[14].

Summary

The literature provides many methods to measure inequality. In addition, they to a varied extent meet the criteria of inequality measure assessment, i.e. the Pigou-Dalton transfer principle, the income scale independence, the principle of population or decomposability. They are based on diverse theoretical foundations that allow to reflect the positive and normative nature of this economic category. Measures based

12 The Dalton index does not meet the income scale independence criterion. With the limitations of the welfare function, including the adoption of a certain value for parameter ε, it can meet the Pigou-Dalton criterion.

13 Equivalent income is an individual income, which - with its equal distribution in the society – equalises the overall welfare with the welfare achieved with a given, i.e. existing distribution of aggregate income.

14 On the basis of the Atkinson index, U. Ebert (1999) presented another measure of inequality. The difference in interpretation of the Ebert and Atkinson indices consists in that the first of them illustrates the loss of welfare resulting from inequality as the percentage of equally distributed equivalent income, while the other one as the percentage of average income for the population.

on the Lorenz curve and generalised entropy are of a more instrumental (positive) nature, while those based on the welfare function – are of normative nature.

However, the use of different inequality measures, especially in comparative studies, depends on the possibility to compare the values of these measures calculated for various countries. Thus, although measures based on the welfare function (Atkinson measures) better than other measures reflect the normative nature of inequality, their basic weakness results from limited international comparability. Calculation of the value of a given measure for a single country entails adoption of a certain value of inequality aversion parameter, which can incorrectly reflect social distribution-related preferences in the other analysed countries. For this reason, such a measure is of little use in comparative studies.

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